Math Olympiad Grading Rubrics and Solutions

Julissa Barahona

November 2025

Grading Philosophy and Emphasis

Clarity of solution and process is paramount in this competition. While correct answers are important, we place significant emphasis on how students communicate their mathematical reasoning.

Key Grading Priorities:

- **Clear, step-by-step logical progression** - Each step should naturally follow from the previous one
- **Justification of key transitions** - Important steps should be explained, not just stated
- **Proper mathematical notation** - Use of appropriate symbols and formalism
- **Organization and readability** - Solutions should be structured for easy understanding
- **Creativity and insight** - Novel approaches or elegant simplifications

Partial credit will be awarded specifically for demonstrating **clear mathematical thinking** even when the final answer is incomplete or contains minor errors. A well-documented process with clear reasoning may earn more points than a correct answer with no explanation.

Remember: The goal is not just to get the right answer, but to demonstrate *how you think* and *why your approach works.* Focus on communicating your reasoning clearly and completely.

1 Overall Grading Framework

1.1 Assessment Criteria

- **Correctness (40%):** Mathematical accuracy and final answer

- **Reasoning (30%):** Logical structure and justification of steps
- **Creativity/Insight (20%):** Novel approaches, elegant methods, or deep mathematical insight
- **Clarity/Presentation (10%):** Organization, notation, and communication

1.2 Creativity and Insight Evaluation

- **Excellent (18-20 points):** Highly original approach, elegant simplification, or deep insight beyond standard solution
- **Good (15-17 points):** Some creative elements or nice insights in approach
- **Basic (10-14 points):** Standard approach with minor creative touches
- **Limited (0-9 points):** Purely standard approach with no creative elements

2 Problem 1: Medicinal/Chemical Mathematics

2.1 Grading Rubric

Points	Criteria	Description
10	**Complete Solution**	Correct equation of state derived with all steps shown, creative insight in approach
8-9	**Excellent**	Correct approach with minor error, shows good mathematical insight
6-7	**Good**	Substantial progress with clear reasoning, some creative elements
4-5	**Partial**	Basic understanding with partial progress, standard approach
2-3	**Minimal**	States relevant formulas but limited progress or creativity
0-1	**No Attempt**	Blank or completely irrelevant

2.2 Sample Solution

Given:

Molar Gibbs energy:

$$G_m = RT \ln p + A + Bp + \frac{1}{2}Cp^2 + \frac{1}{3}Dp^3$$

Key Steps for Partial Credit:

Step 1: Recall fundamental thermodynamic relation (3 points)

$$V_m = \left(\frac{\partial G_m}{\partial p}\right)_T$$

Step 2: Differentiate the Gibbs energy (3 points)

$$\begin{aligned}\frac{\partial G_m}{\partial p} &= \frac{\partial}{\partial p}\left(RT\ln p + A + Bp + \frac{1}{2}Cp^2 + \frac{1}{3}Dp^3\right) \\ &= \frac{RT}{p} + B + Cp + Dp^2\end{aligned}$$

Step 3: Equate to molar volume (2 points)

$$V_m = \frac{RT}{p} + B + Cp + Dp^2$$

Step 4: Rearrange to equation of state (2 points)

$$pV_m = RT + Bp + Cp^2 + Dp^3$$

Final Answer:

$$\boxed{pV_m = RT + Bp + Cp^2 + Dp^3}$$

3 Problem 2: Mathematical Economics

3.1 Grading Rubric

Points	Criteria	Description
10	**Complete Proof**	Correct derivation with elegant method and clear explanation
8-9	**Excellent**	Correct approach with minor error, creative use of symmetry or efficiency
6-7	**Good**	Substantial progress with clear reasoning, standard approach
4-5	**Partial**	Basic setup with some progress, limited creativity
2-3	**Minimal**	States given information without meaningful application
0-1	**No Attempt**	Blank or completely irrelevant

3.2 Sample Solution

Given:

Utility functions: $U_1 = q_{11}q_{12}$, $U_2 = q_{21}q_{22}$
Resource constraints: $q_{11} + q_{21} = q_1$, $q_{12} + q_{22} = q_2$
Efficiency condition: $\frac{q_{12}}{q_{11}} = \frac{q_{22}}{q_{21}}$

Key Steps for Partial Credit:

Step 1: Use efficiency condition (2 points) Let $\frac{q_{12}}{q_{11}} = \frac{q_{22}}{q_{21}} = k$

Step 2: Express utilities in terms of allocations (3 points)

$$U_1^0 = q_{11}q_{12} = q_{11}(kq_{11}) = kq_{11}^2$$
$$U_2^0 = q_{21}q_{22} = q_{21}(kq_{21}) = kq_{21}^2$$

Step 3: Solve for allocations (3 points)

$$q_{11} = \sqrt{\frac{U_1^0}{k}}, \quad q_{21} = \sqrt{\frac{U_2^0}{k}}$$
$$q_{12} = kq_{11} = \sqrt{kU_1^0}, \quad q_{22} = kq_{21} = \sqrt{kU_2^0}$$

Step 4: Apply resource constraints and derive final result (2 points)

$$q_1 = q_{11} + q_{21} = \frac{1}{\sqrt{k}}(\sqrt{U_1^0} + \sqrt{U_2^0})$$
$$q_2 = q_{12} + q_{22} = \sqrt{k}(\sqrt{U_1^0} + \sqrt{U_2^0})$$
$$q_1q_2 = (\sqrt{U_1^0} + \sqrt{U_2^0})^2$$

Final Answer:

$$\boxed{q_1q_2 = \left(\sqrt{U_1^0} + \sqrt{U_2^0}\right)^2}$$

4 Problem 3: Mathematical Computer Science

4.1 Grading Rubric

Points	Criteria	Description
9-10	**Excellent**	Both parts correct with elegant reasoning and creative insights
7-8	**Good**	Both parts correct with clear reasoning, standard approach
5-6	**Satisfactory**	One part fully correct, other partially correct
3-4	**Partial**	Basic understanding with limited progress in both parts
1-2	**Minimal**	Some relevant concepts but major errors
0	**No Attempt**	Blank or completely irrelevant

4.2 Sample Solution

Part (a): Independent Random Routing

Key Steps for Partial Credit:

Step 1: Define indicator variables (2 points) Let I_0, I_1, I_2 be indicator variables for packets from inputs 0, 1, 2 crossing the dashed edge.

Step 2: Calculate individual expectations (2 points) By symmetry, each input has equal probability to route to any output:

$$\mathbb{E}[I_0] = \mathbb{E}[I_1] = \mathbb{E}[I_2] = \frac{1}{3}$$

Step 3: Expectation of sum (1 point)

$$\mathbb{E}[T] = \mathbb{E}[I_0 + I_1 + I_2] = 3 \times \frac{1}{3} = 1$$

Step 4: Variance calculation (3 points) For indicator variables: $\text{Var}(I_i) = p(1-p) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
Since destinations are independent: $\text{Cov}(I_i, I_j) = 0$ for $i \neq j$

$$\text{Var}(T) = 3 \times \frac{2}{9} = \frac{2}{3}$$

Part (b): Permutation Routing

Key Steps for Partial Credit:

Step 1: Permutation constraint (2 points) Each input sends to a distinct output, exactly one permutation chosen uniformly.

Step 2: Symmetry argument (2 points) By symmetry of the network, each of the three possible paths through the dashed edge is equally likely to be used.

Step 3: Expected value (1 point) Since exactly one packet crosses the dashed edge in every permutation:

$$\mathbb{E}[T] = 1$$

Final Answers:

$$\text{(a)} \quad \boxed{\mathbb{E}[T] = 1, \quad \text{Var}(T) = \frac{2}{3}}$$

$$\text{(b)} \quad \boxed{\mathbb{E}[T] = 1}$$

5 Problem 4: Mathematical Physics

5.1 Grading Rubric

Points	Criteria	Description
9-10	**Excellent**	Both parts correct with creative physical insight and elegant method
7-8	**Good**	Both parts correct with clear reasoning and good physical intuition
5-6	**Satisfactory**	One part fully correct, other partially correct with standard approach
3-4	**Partial**	Basic setup with limited progress in both parts
1-2	**Minimal**	Some relevant formulas but major conceptual errors
0	**No Attempt**	Blank or completely irrelevant

5.2 Sample Solution

Given:

Masses: m and $4m$, Radii: a and $2a$, Rod length: $l = \sqrt{24}a$

Part (a): Center of Mass Rotation

Key Steps for Partial Credit:

Step 1: Rolling without slipping condition (2 points)

$$v = \omega r \quad \text{for each disc}$$

Step 2: Relate angular speeds (1 point) Let ω_1 and ω_2 be angular speeds of discs about their centers. Since they're rigidly connected: $\omega_1 a = \omega_2(2a) = \omega_{CM} R$

Step 3: Find center of mass position (1 point)

$$R = \frac{m \cdot 0 + 4m \cdot l}{m + 4m} = \frac{4}{5}l = \frac{4}{5}\sqrt{24}a$$

Step 4: Solve for ω_{CM} (1 point) From $\omega a = \omega_{CM} R$:

$$\omega_{CM} = \frac{\omega a}{R} = \frac{\omega a}{\frac{4}{5}\sqrt{24}a} = \frac{5\omega}{4\sqrt{24}} = \frac{\omega}{5}$$

Part (b): Angular Momentum about Center of Mass

Key Steps for Partial Credit:

Step 1: Calculate moments of inertia (2 points)

$$I_1 = \frac{1}{2}ma^2$$
$$I_2 = \frac{1}{2}(4m)(2a)^2 = 8ma^2$$

Step 2: Total moment of inertia about CM (2 points) Since CM is on the line joining centers:

$$I_{CM} = I_1 + I_2 = \frac{1}{2}ma^2 + 8ma^2 = \frac{17}{2}ma^2$$

Step 3: Angular momentum about CM (1 point)

$$L_{CM} = I_{CM}\omega = \frac{17}{2}ma^2\omega$$

Final Answers:

$$\text{(a)} \quad \boxed{\omega_{CM} = \frac{\omega}{5}}$$

$$\text{(b)} \quad \boxed{L_{CM} = \frac{17}{2}ma^2\omega}$$

6 Problem 5: Pure Mathematics

6.1 Grading Rubric

Points	Criteria	Description
9-10	**Excellent**	Complete solution with elegant proof and creative insights into polynomial structure
7-8	**Good**	Complete solution with clear reasoning, recognizes complex number connection
5-6	**Satisfactory**	Finds main polynomials but incomplete classification, standard approach
3-4	**Partial**	Identifies some solutions with limited reasoning
1-2	**Minimal**	Tests simple cases without meaningful progress
0	**No Attempt**	Blank or completely irrelevant

6.2 Sample Solution

Given:

Find all polynomials $P(x, y)$ with real coefficients satisfying:

$$P(x, y)P(z, t) = P(xz - yt, xt + yz)$$

Key Steps for Partial Credit:

Step 1: Test constant polynomials (2 points) If $P(x, y) = c$, then:

$$c^2 = c \Rightarrow c(c - 1) = 0 \Rightarrow c = 0 \text{ or } c = 1$$

So $P(x, y) = 0$ and $P(x, y) = 1$ are solutions.

Step 2: Recognize complex number structure (2 points) Let $z = x + iy$, $w = z + it$, then:

$$(x + iy)(z + it) = (xz - yt) + i(xt + yz)$$

So the identity becomes:

$$P(x, y)P(z, t) = P(\Re(zw), \Im(zw))$$

Step 3: Test $P(x, y) = x^2 + y^2$ (3 points) Left side: $(x^2 + y^2)(z^2 + t^2)$
Right side: $(xz - yt)^2 + (xt + yz)^2 = x^2z^2 + y^2t^2 + x^2t^2 + y^2z^2$
Both equal $x^2z^2 + x^2t^2 + y^2z^2 + y^2t^2$, so identity holds.

Step 4: Test higher powers (2 points) Check $P(x, y) = (x^2 + y^2)^n$:

$$\begin{aligned}
\text{LHS} &= (x^2 + y^2)^n(z^2 + t^2)^n \\
\text{RHS} &= ((xz - yt)^2 + (xt + yz)^2)^n = (x^2z^2 + y^2t^2 + x^2t^2 + y^2z^2)^n \\
&= (x^2 + y^2)^n(z^2 + t^2)^n
\end{aligned}$$

So all these work.

Step 5: Show these are all solutions (1 point) If P is not identically zero, let $P(1, 0) = a \neq 0$.
Setting $y = t = 0$ gives: $P(x, 0)P(z, 0) = P(xz, 0)$
So $Q(x) = P(x, 0)$ satisfies $Q(x)Q(z) = Q(xz)$, implying $Q(x) = x^n$ for some n.
By analyzing the identity structure, one can show $P(x, y) = (x^2 + y^2)^n$.

Final Answer:

$$\boxed{P(x, y) = 0, \quad P(x, y) = 1, \quad P(x, y) = (x^2 + y^2)^n \text{ for } n \in \mathbb{N}}$$

7 Summary of Partial Credit Allocation

Problem	Key Steps	Points per Step
1	4 steps	3, 3, 2, 2
2	4 steps	2, 3, 3, 2
3a	4 steps	2, 2, 1, 3
3b	3 steps	2, 2, 1
4a	4 steps	2, 1, 1, 1
4b	3 steps	2, 2, 1
5	5 steps	2, 2, 3, 2, 1

Score Sheet and Final Grading

Problem	Max Points	Points Awarded
1. Medicinal/Chemical Mathematics	10	______
2. Mathematical Economics	10	______
3. Mathematical Computer Science	10	______
4. Mathematical Physics	10	______
5. Pure Mathematics	10	______
Grand Total	**50**	______

Creativity/Insight Assessment:

- Problem 1: ____/2 (Integrated into score above)
- Problem 2: ____/2 (Integrated into score above)
- Problem 3: ____/2 (Integrated into score above)
- Problem 4: ____/2 (Integrated into score above)
- Problem 5: ____/2 (Integrated into score above)

Final Score: ________ / 50

Grader's Comments:

Overall Assessment:
Mathematical Creativity and Insight:
Key Strengths:
Areas for Improvement:

Grader's Signature: ______________________ **Date:** ______________